Determination and Prediction of Orbital Parameters of the Radioastron Mission

M. Zakhvatkin [1], V. Stepanyants [1], A. Tuchin [1], Yu. Ponomarev [2]

[1]Keldysh Institute of Applied Mathematics (KIAM)
[2]Astrospace Center of Lebedev Physical Institute (ASC)

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Radioastron mission

Orbit

Launch date:	18 July 2011
Perigee altitude:	1000 – 67 000 km
Apogee distance:	up to 370 000 km
Period:	8–9 days

Mission

Purpose:	VLBI observations
Bands:	P, L, C, K
Data channel:	2x72 Mbps

Orbit knowledge is required for the interferometric data correlation.

Main perturbations

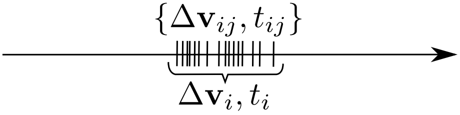
Nature	Maximum, m/s^2	Average, m/s^2
Spherical harmonics	$3.8{\cdot}10^{-3}$	$3.3{\cdot}10^{-6}$
Third bodies	$2.3{\cdot}10^{-4}$	$4.1{\cdot}10^{-5}$
Direct solar radiation	$1.9{\cdot}10^{-7}$	$1.5{\cdot}10^{-7}$
Unloadings	$5.8{\cdot}10^{-8}$	$5.8{\cdot}10^{-8}$
Tides	$6.6{\cdot}10^{-8}$	$2.3{\cdot}10^{-11}$
Earth radiation	$2.1{\cdot}10^{-8}$	$1.1{\cdot}10^{-10}$

The satellite is not equipped with accelerometers.

Unloadings of reaction wheels

An unloading consists of dozens of firings, j-th firing of the i-th unloading provides $\Delta \mathbf{v}_{ij} = \dfrac{\Delta m_{ij} I(\tau_{ij})}{M} \mathbf{e}_{ij}$ at t_{ij}

All firings of the unloading are summed up into one impulse $\Delta \mathbf{v}_i$ applied at weighted time t_i.



Weighted time and the estimate of the impulse are as follows

$$t_i = \frac{\sum_j v_{ij} t_{ij}}{\sum_j v_{ij}} \quad \mathbf{v}_i^0 = \sum_j \mathbf{v}_{ij}(\Delta m_{ij}, \tau_{ij}, \mathbf{e}_{ij}).$$

The perturbation due to unloadings on the interval of interest is described with the set of impulses $\{\Delta \mathbf{v}_i, t_i\}$.

Direct solar radiation pressure

Decomposition of the solar radiation impacting a flat surface

$$\mathbf{F} = (1-\alpha)\mathbf{F}_a + \alpha\mu\mathbf{F}_s + \alpha(1-\mu)\mathbf{F}_d$$

$\alpha \in [0,1]$ — reflectivity,

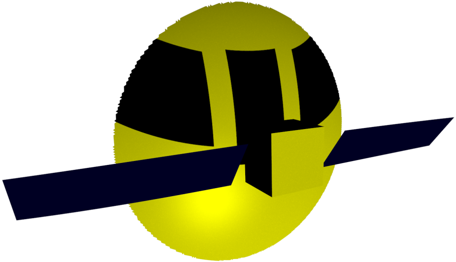
$\mu \in [0,1]$ — specularity.

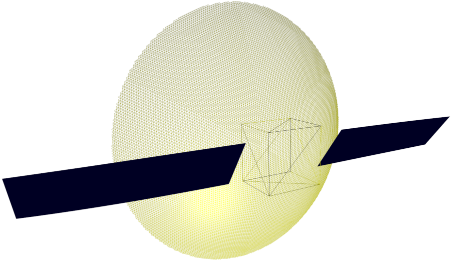
allows to represent net SRP force and torque as functions of parameters α_i and μ_i

$$\mathbf{F}_{SRP} = \sum_{i=1}^{N} \eta_i \mathbf{F}(A_i, \mathbf{s}, \mathbf{n}_i, \alpha_i, \mu_i),$$

$$\mathbf{M}_{SRP} = \sum_{i=1}^{N} \eta_i \mathbf{r_i} \times \mathbf{F}(A_i, \mathbf{s}, \mathbf{n}_i, \alpha_i, \mu_i),$$

The satellite structure





Element	Surface	Coefficients
space radio telescope	reflecting (MLI)	α_1, μ_1
spacecraft bus	reflecting (MLI)	α_1, μ_1
solar panels	absorbing	α_2 ($\mu_2 = 1$)

specularity coefficient of solar panels is fixed to avoid strong correlation with α_2

Propagation

passing through an unloading:

$$(t_{i-0}, \mathbf{r}(t_{i-0}), \mathbf{v}(t_{i-0}), \ldots) \to (t_{i+0}, \mathbf{r}(t_{i+0}), \mathbf{v}(t_{i-0}) + \Delta\mathbf{v}_i, \ldots),$$
$$(t_{i-0}, \mathbf{r}(t_{i-0}), \mathbf{v}(t_{i+0}) - \Delta\mathbf{v}_i, \ldots) \leftarrow (t_{i+0}, \mathbf{r}(t_{i+0}), \mathbf{v}(t_{i+0}), \ldots).$$

Gravity field	EGM96
Third bodies	DE-405
Tides	IERS 2003 convention
Direct SRP	parameterized with α_1, μ_1 and α_2
Earth radiation	18x9 constant coeff.

Motion of the center of mass is determined by :

$$\mathbf{X}_0(t_0), \alpha_1, \mu_1, \alpha_2, \Delta\mathbf{v}_1, \ldots, \Delta\mathbf{v}_n$$

Observations

Radio

- Two-way range, two-way Doppler
- One-way Doppler

System	Band	D	$\dot{D}$	$\dot{D}_{1w}$
Ussuriysk RT-70, "Klen-D"	C	✓	✓	
Ussuriysk RT-70, "Phobos"	X			✓
Bear Lakes RT-64, "Cobalt-M"	C	✓	✓	
Bear Lakes RT-64, "Cortex"	X			✓
Puschino RT-22	X, Ku			✓
Green Bank, 140ft	X, Ku			✓

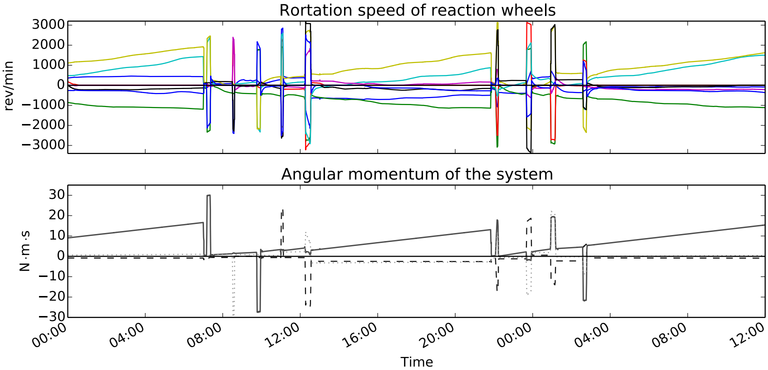
Optical

- CCD RA/Dec: ISON, ASC

Telemetry

- Observed impulses of unloadings $\Delta\mathbf{v}_i^0$.
- Observed torque $\mathbf{M}$.

Perturbing torque observations



During constant attitude far from the Earth

$$\sum_{i=1}^{8} \mathbf{a_i} I_i(\Omega_i(t_2) - \Omega_i(t_1)) = \mathbf{M}_{SRP}(\Lambda, \alpha_1, \mu_1, \alpha_2)(t_2 - t_1).$$

Introduce the following difference of observed and computed torque

$$\boldsymbol{\zeta} = \sum_{i=1}^{8} \frac{\mathbf{a_i} I_i \left[\Omega_i(t_2) - \Omega_i(t_1)\right]}{t_2 - t_1} - \mathbf{M}_{SRP}(\Lambda, \alpha_1, \mu_1, \alpha_2).$$

Orbit determination

Solve for the following parameters on the interval $[t_b, t_e]$

$$\mathbf{Q} = \{\mathbf{X_0}(t_0), \alpha_1, \mu_1, \alpha_2, \Delta\mathbf{v}_1, \ldots, \Delta\mathbf{v}_n\}.$$

using tracking data

$$\mathbf{\Psi} = \{\mathbf{D}, \dot{\mathbf{D}}, \dot{\mathbf{D}}_{1w}, \boldsymbol{\alpha}, \boldsymbol{\delta}\}$$

and on-board observations

$$\{\mathbf{\Omega}(t), \Delta\mathbf{v}_1^0, \ldots, \Delta\mathbf{v}_n^0\}$$

to minimize the functional

$$\Phi = (\mathbf{\Psi}_o - \mathbf{\Psi}_c)^{\mathsf{T}}\mathbf{P}(\mathbf{\Psi}_o - \mathbf{\Psi}_c) + \sum_{j=1}^{N} \boldsymbol{\zeta}_j^{\mathsf{T}}\mathbf{P}_j^{sp}\boldsymbol{\zeta}_j +$$

$$+ \sum_{i=1}^{n} (\Delta\mathbf{v}_i^0 - \Delta\mathbf{v}_i)^{\mathsf{T}}\mathbf{P}_i(\Delta\mathbf{v}_i^0 - \Delta\mathbf{v}_i),$$

Orbit determination

Two intervals have been considered:

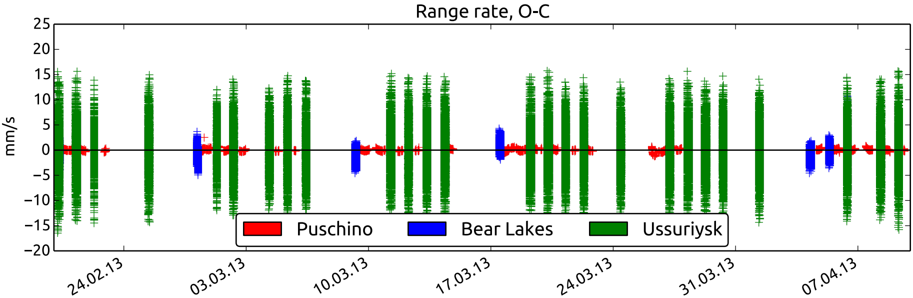
- 20-Feb-2013 – 10-Apr-2013 (Int. 1)
- 10-Apr-2013 – 30-May-2013 (Int. 2)

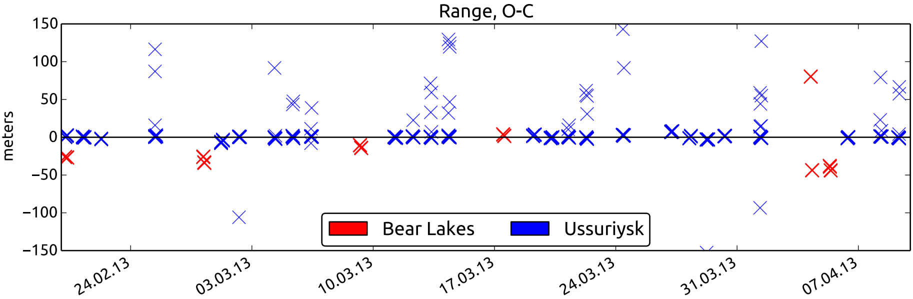
Several models were used :

- Simple SRP, No unloadings.
- Simple SRP, unloadings fixed on their nominal values $\Delta\mathbf{v}_i^0$
- SRP depends on three coefficients, unloadings fixed on their nominal values $\Delta\mathbf{v}_i^0$
- SRP depends on three coefficients, unloadings are solved for.

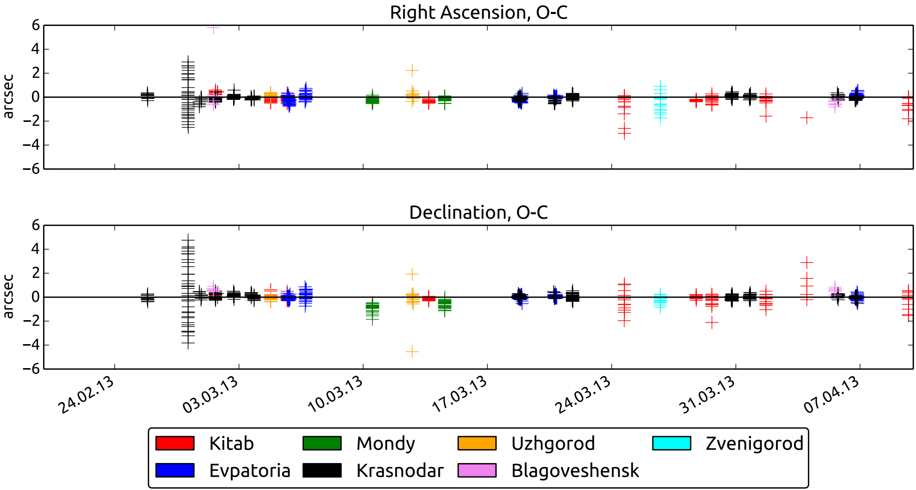
Parameters obtained on the Int. 1 will be used for orbit prediction on the Int. 2.

Radioastron D, $\dot{D}$, 20.02.13 – 10.04.13

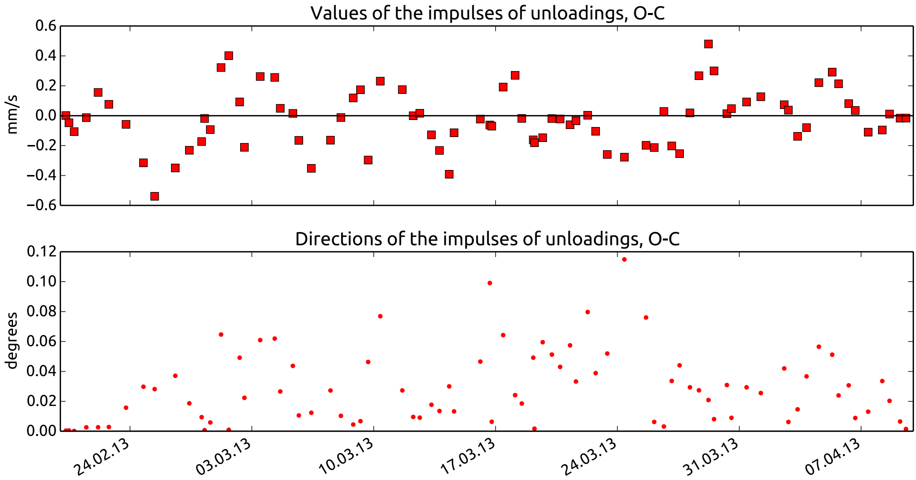




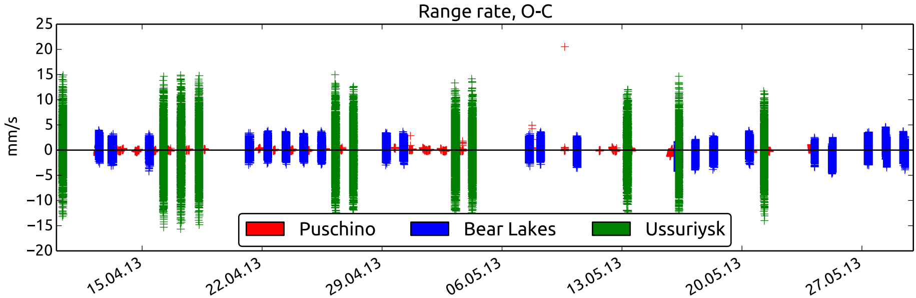
Radioastron (α, δ), 20.02.13 – 10.04.13

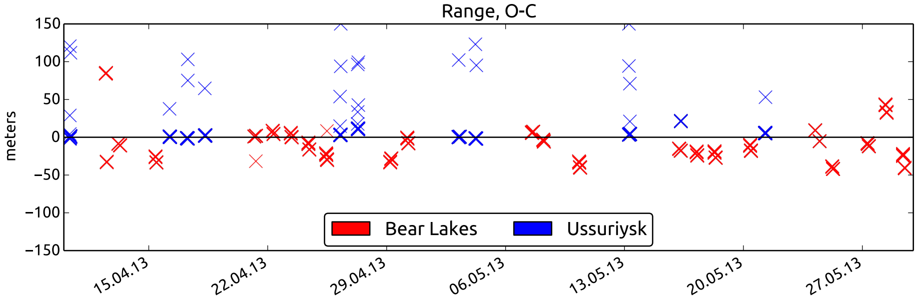


Radioastron $\{\Delta\mathbf{v}_i^0 - \Delta\mathbf{v}_i\}$, 20.02.13 – 10.04.13

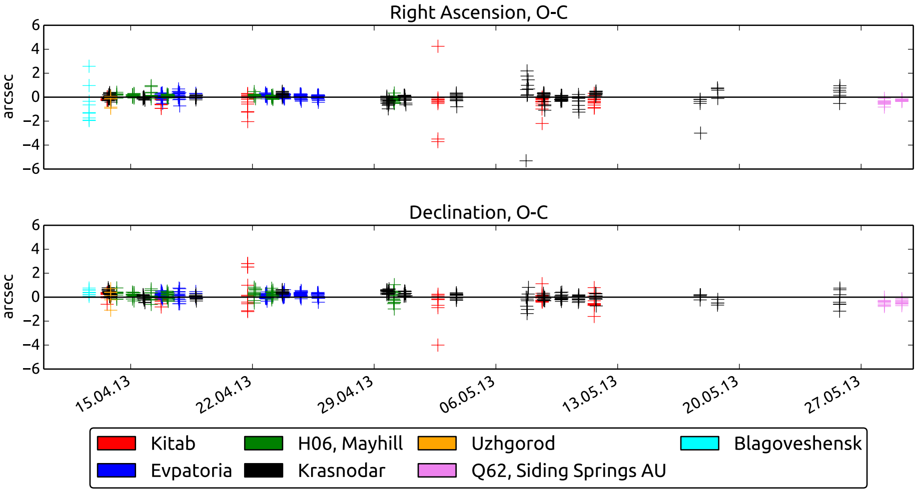


Radioastron D, $\dot{D}$, 10.04.13 – 30.05.13

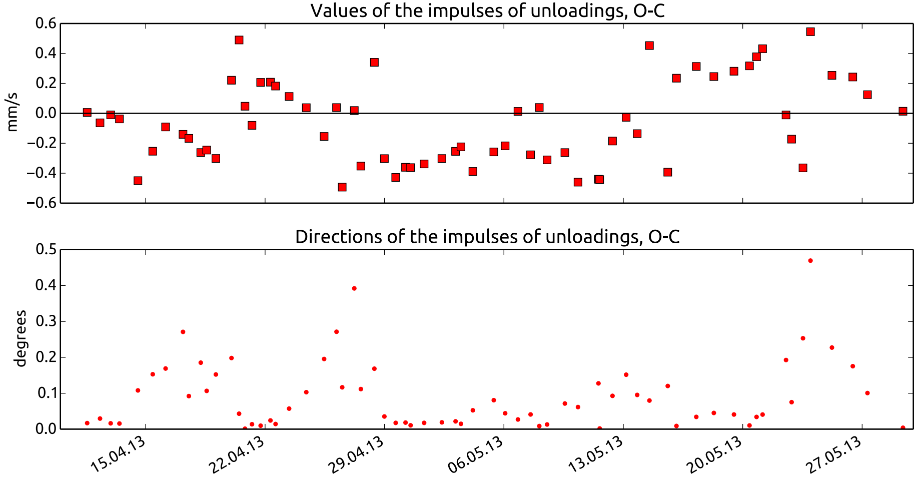




Radioastron (α, δ), 10.04.13 – 30.05.13



Radioastron $\{\Delta\mathbf{v}_i^0 - \mathbf{v}_i\}$, 10.04.13 – 30.05.13



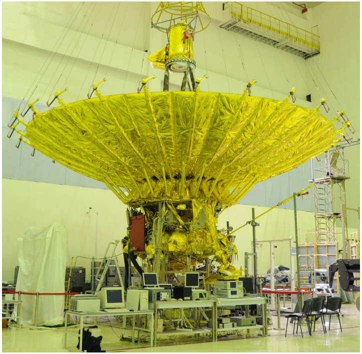
Orbit determination results

Dimensionless standard deviation: 20.02.13 – 10.04.13 (Int. 1) and 10.04.13 – 30.05.13 (Int. 2)

№	SRP model	Unloadings	σ_1	σ_2	Δr, km	Δv, mm/s
1	Simple, 1 coeff.	Not considered	12.43677	9.18588	71.71	288.1
2	Simple, 1 coeff.	Nominal	4.72914	6.78832	36.76	113.3
3	Proposed, 3 coeff.	Nominal	1.20896	0.63767	7.57	8.9
4	Proposed, 3 coeff.	Solved for	0.28198	0.24907	0.21	2.3

Estimated solar radiation pressure coefficients

Parameter	Int. 1	Int. 2
α_1	0.754	0.791
μ_1	0.087	0.089
α_2	0.063	0.102



Orbit prediction

Necessary elements

- Attitude forecast (observation schedule + service attitude)

$$(\Lambda_1, t_1, t_1'), (\Lambda_2, t_2, t_2'), \ldots (\Lambda_n, t_n, t_n').$$

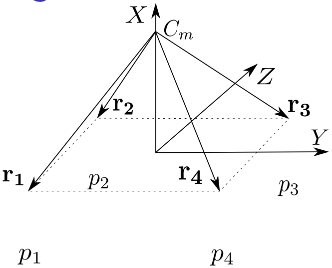
  With estimated SRP coefficients determines corresponding perturbation and accumulation of angular momentum by the reaction wheels

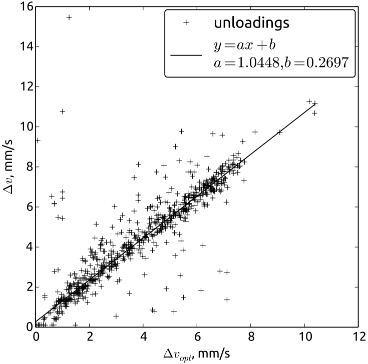
- Conversion of accumulated angular momentum to impulses of unloadings

$$\mathbf{K}(t) \xrightarrow{\Delta\mathbf{v}(t)} \mathbf{K}(t + \delta t)$$

- Prediction of times of occurrence of unloadings.

Angular momentum to unloading





Angular momentum changing during an unloading can be described as follows:

$$\sum_{i=1}^{8} \mathbf{a_i} I_i \Omega_i(t_u) = \sum_{j=1}^{4} \mathbf{r_j} \times \mathbf{e_j} p_j,$$

- an unloading takes relatively short time,
- reaction wheels stop,
- the satellite is not rotating.

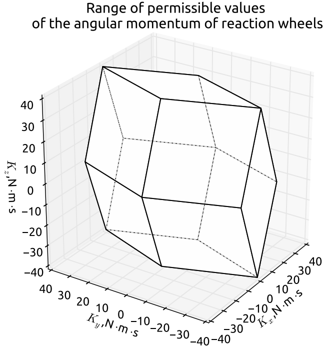
where $p_j \geq 0$ are the propellant momenta. The equation can be resolved with respect to $\{p_j\}$ with additional condition:

$$\sum_j p_j \to \min .$$

An impulse of an unloading

$$\Delta \mathbf{v}^*(\mathbf{K}) = -\frac{\sum_i p_i \mathbf{e_i}}{M}, \qquad \Delta \mathbf{v} = \Delta \mathbf{v}(\Delta \mathbf{v}^*).$$

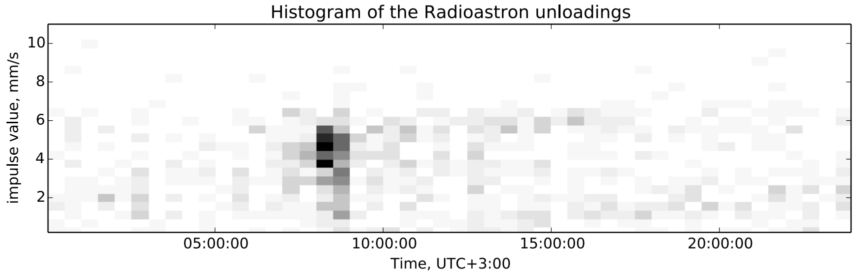
Prediction of the time of next unloading



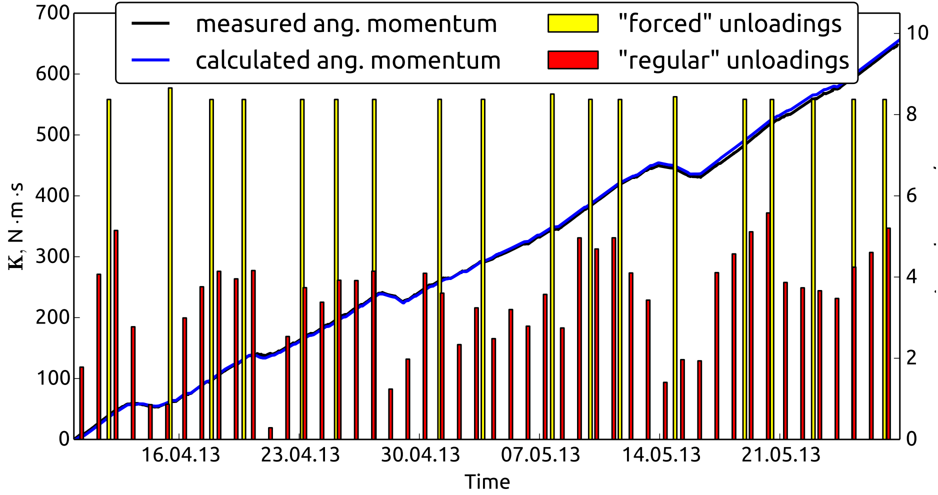
- An unloading should be conducted if accumulated angular momentum is too high $\mathbf{K}(t) \notin U$

$$U = \{\mathbf{K} = \sum_{i=1}^{N} \mathbf{a}_i I_i \Omega_i : |\Omega_i| \le \Omega_{max}, i = \overline{1, N}\}.$$

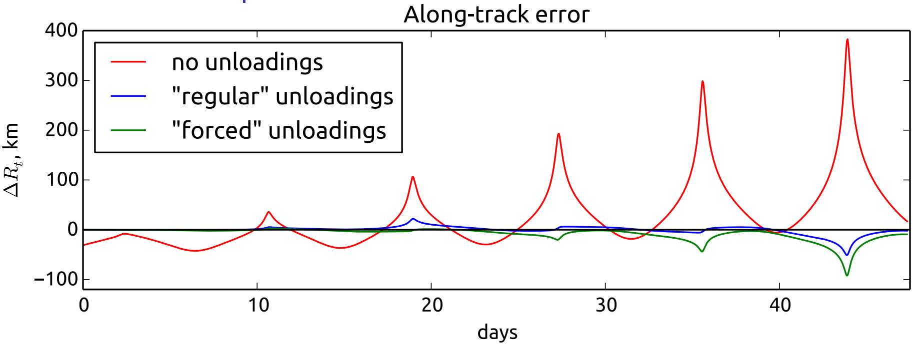
- Unloadings can be conducted on daily basis in the same time

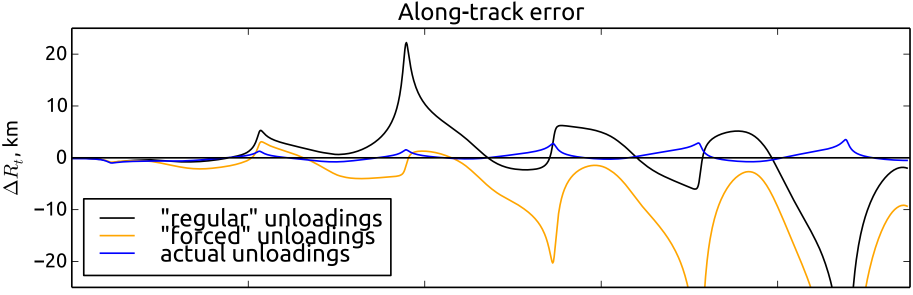


Prediction of unloadings on the Int. 2 (10-Apr-2013 – 30-May-2013)



Results of orbit prediction





Summary

- Adjustable Radioastron SRP model was developed and tested.
- Parameters of the SRP model was estimated by using both motion of the center of mass and motion around the center of mass.
- Determined orbits are successfully used for correlation of the Radioastron observations.
- An unloading prediction approach, important for future Sun-Earth L_2 missions (Spectr-R,'Millimetron) based on the same platform, was tested on the Radioastron data.

Thank you for your attention!